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On the relativistic invariance of the quantization in the meson pair theory

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In some papers [1], [2] the authors have investigated the possible charged meson pair theories, in which the mesons are assumed to have spin 0 or 1. In these papers we used the generalized Dirac equation for the mesons, and formulated the possible Lorentz invariant pair interactions with nucleons, expressed in the β_k:s of the mesons and the γ_k:s of the nucleons.

In the present paper we intend to investigate the Lorentz invariance of the quantization of the theory. This question has been considered for some special cases of the symmetrical meson pair theory in a preliminary note by Kanesawa [3], using Tomonaga's method of field quantization. Due to the fact, that the Dirac equation of the mesons does not contain the time derivatives of the so-called redundant components, the corresponding components of the Dirac equation cannot be deduced from a Hamilton formalism. Thus the Dirac equation, transformed to Tomonaga's "interaction representation", will not in general be of the unperturbed form. Kanesawa now tackles this difficulty by introducing new redundant components in the interaction representation, so chosen, that the Dirac equation becomes unperturbed. Such a method seems, however, to be rather complicated for more general interactions. For this reason we have tried another method, similar to that proposed by Heisenberg and Pauli [4] for the electromagnetic field. They made use of small additional terms in order to introduce a conjugate momentum to the scalar potential. In the same manner we introduce additional terms in the generalized Dirac equation of the mesons, thus making all components non-redundant. The relativistic invariance of the quantization now follows immediately from the proof given by Heisenberg and Pauli [4]. Further, all the components of the Dirac equation being now derivable from a Hamilton formalism, we get an unperturbed Dirac equation in the interaction representation. Hence, the proof of the relativistic invariance can also be performed without difficulty according to the method of Tomonaga [5].

The invariance proof, sketched above, is performed in the following manner: We introduce the invariant Lagrangian

$$L = -i c \tilde{\Psi} \left\{ \sum_{k=1}^{4} (\beta_k + \varepsilon_k) \left(-i \hbar \frac{\partial}{\partial x_k} \right) + i \mu c \right\} \Psi -$$

$$- i c \tilde{\psi} \left\{ \sum_{k=1}^{4} \gamma_k \left(-i \hbar \frac{\partial}{\partial x_k} \right) + i M c \right\} \psi + i c \tilde{\Psi} \tilde{\psi} I \psi \Psi .$$

Here Ψ is the meson field and ψ the field of the nucleons. I is an invariant in the β_k:*s* and γ_k:s, and ε_k is a four-vector, constant in time and space, commuting with the β_k:s and γ_k:s. The conjugate momenta to the independent field variables Ψ_λ are defined as $P_{\lambda 4}$, when

$$P_{\lambda k} = \frac{1}{i c} \frac{\partial L}{\partial \dfrac{\partial \Psi_\lambda}{\partial x_k}} = i \hbar \{\tilde{\Psi} (\beta_k + \varepsilon_k)\}_\lambda,$$

and correspondingly for the conjugate momenta $\pi_{\lambda 4}$ of the ψ_λ:s. We can now form the Hamiltonian in the ordinary way:

$$H = i c \tilde{\Psi} \left\{ \sum_{k=1}^{3} (\beta_k + \varepsilon_k) \left(- i \hbar \frac{\partial}{\partial x_k} \right) + i \mu c \right\} \Psi +$$

$$+ i c \tilde{\psi} \left\{ \sum_{k=1}^{3} \gamma_k \left(- i \hbar \frac{\partial}{\partial x_k} \right) + i M c \right\} \psi - i c \tilde{\Psi} \tilde{\psi} I \psi \Psi,$$

where the $\tilde{\Psi}_\lambda$:s and $\tilde{\psi}_\lambda$:s can easily be expressed in the $P_{\lambda 4}$:s and $\pi_{\lambda 4}$:s respectively. The commutation rules are:

$$[\Psi_\lambda (\boldsymbol{r}'), P_{\mu 4} (\boldsymbol{r}'')] = i \hbar \, \delta_{\lambda \mu} \, \delta (\boldsymbol{r}' - \boldsymbol{r}'')$$

and

$$[\psi_\lambda (\boldsymbol{r}'), \pi_{\mu 4} (\boldsymbol{r}'')]_+ = i \hbar \, \delta_{\lambda \mu} \, \delta (\boldsymbol{r}' - \boldsymbol{r}'').$$

With these commutation rules HAMILTON's equations give:

$$\left\{ \sum_{k=1}^{4} (\beta_k + \varepsilon_k) \left(- i \hbar \frac{\partial}{\partial x_k} \right) + i \mu c - \tilde{\psi} I \psi \right\} \Psi = 0$$

and

$$\left\{ \sum_{k=1}^{4} \gamma_k \left(- i \hbar \frac{\partial}{\partial x_k} \right) + i M c - \tilde{\Psi} I \Psi \right\} \psi = 0.$$

The HAMILTON formalism here developed is easily seen to conform to the conditions for the LORENTZ invariance of the quantization assumed by HEISENBERG and PAULI [4].

With the quantization used in [1] ch. VI the energy of the meson field, when the nucleons are considered as fixed sources, is equal to half the sum of the moduli of all the energy eigenvalues of the non-quantized meson equation. Whith the method used in this paper the energy of the meson field is obtained in an analogous way. Now, when the ε_k:s are introduced, the original eigenvalues are continually altered. They can be calculated by means of a perturbation calculation similar to that in [2] ch. II. Half the sum of the moduli of these eigenvalues is for $\varepsilon_k = 0$ the same as the result of [2] ch. II. In addition to these eigenvalues, however, we obtain for $\varepsilon_k \neq 0$ new eigenvalues, tending to infinity for $\varepsilon_k = 0$. It is easily seen that the contribution of these eigenvalues to the result of the perturbation calculation vanishes with ε_k. Thus the final result of the procedure of this paper, which is evidently LORENTZ invariant, is the same as the result of [1] and [2].

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